Key

Math 4

Name _____

Date____

4-4 Trigonometric Proofs

In this Activity, you will be working towards the following learning goals:
I can use and define the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent
I can use the fundamental trigonometric identities to simplify expressions and verify equivalences

Useful Trigonometric Identities:

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Helpful hints in case you have an identity crisis:

- Change everything to sine and/or cosine
- Look for occurrences of the basic three identities
- Factor
- Combine +/- expressions into a single fraction using a common denominator
- $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$
- Use "fancy version of 1", for example multiply by $\frac{\tan x}{\tan x}$
- Multiply by the conjugate
- $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

Trigonometric Proof Advice:
Start with the MORE COMPLICATED side and work VERTICALLY just on that side to make it look like the other side. SHOW ALL WORK. *Note: We cannot assume that the equation is true, so properties of equality cannot be applied – i.e. we cannot add, subtract, multiply or divide on both sides of the equation.*

Prove algebraically: $\cos x \tan x = \sin x$

$= \cos x \cdot \frac{\sin x}{\cos x}$

$= \sin x$ ✓

Examples: Prove the following.

1.) $1+\tan^2 x = \sec^2 x$ → $\frac{1}{\cos^2 x}$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$ OR → $= \frac{1}{\cos^2 x}(\cos^2 x + \sin^2 x)$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ $= \frac{1}{\cos^2 x}(1)$

$= \frac{1}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$

$= \sec^2 x$ ✓ $= \sec^2 x$ ✓

2.) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$ ✓

3.) $\cot x \bullet \sin x \bullet \sec x = \cos^2 x + \sin^2 x$

$= \frac{\cos x}{\sin x} \cdot \sin x \cdot \frac{1}{\cos x}$

$= 1$

$= \cos^2 x + \sin^2 x$ ✓

4.) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

$= \frac{(1-\cos x) + (1+\cos x)}{(1-\cos x)(1+\cos x)}$

$= \frac{2}{1+\cos x - \cos x - \cos^2 x}$

$= \frac{2}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x} = 2\csc^2 x$ ✓

5.) $\dfrac{1}{\cot x + \csc x} = \csc x - \cot x$

$= \dfrac{\csc^2 x - \cot^2 x}{\cot x + \csc x}$

$= \dfrac{(\csc x - \cot x)(\csc x + \cot x)}{\cot x + \csc x}$

$= \csc x - \cot x$ ✓